Example

Find the radius \( R \) and the interval of convergence for the series

\[
\sum_{n=0}^{\infty} (2n)!(3x)^n.
\]

Do not check convergence at the endpoints of intervals.

We first compute \( L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| \) for each fixed value of \( x \), and then determine the values of \( x \) for which \( L < 1 \).

Note that

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
= \lim_{n \to \infty} \left| \frac{(2n + 2)!(3x)^{n+1}}{(2n)!(3x)^n} \right|
= \lim_{n \to \infty} |(2n + 2)(2n + 1)3x|
= \infty
\]

for all values of \( x \neq 0 \). If \( x = 0 \), then the terms of the series are identically zero. Thus the radius of convergence is \( R = 0 \) and the series converges only at the point \{0\}.