Example

Find the interval of convergence for the series

\[ \sum_{k=1}^{\infty} (2k)!(3x)^k. \]

Determine whether endpoints (if any) should be included in the interval of convergence.

We first compute \( \bar{\lambda} = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| \) for each fixed value of \( x \), and then determine the values of \( x \) for which \( \bar{\lambda} < 1 \).

Note that

\[
\bar{\lambda} = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right|
= \lim_{k \to \infty} \left| \frac{(2k + 2)(3x)^{k+1}}{(2k)!(3x)^k} \right|
= \lim_{k \to \infty} \left| (2k + 2)(2k + 1)3x \right|
= \infty
\]

for all values of \( x \neq 0 \). If \( x = 0 \), then the terms of the series are identically zero. Thus the series converges only at the point \( \{0\} \).

Note that there is no need to test endpoints in this case, so the interval of convergence is just one point \( \{0\} \).