### Example

Find the radius $R$ and the interval of convergence for the series

$$
\sum_{n=0}^{\infty} (-1)^n (2x + 5)^{2n}.
$$

Do not check convergence at the endpoints of intervals.

We first compute

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
$$

$$
= \lim_{n \to \infty} \left| \frac{(-1)^{n+1}(2x + 5)^{2n+2}}{(-1)^n(2x + 5)^{2n}} \right|
$$

$$
= \lim_{n \to \infty} \left| (-1)(2x + 5)^2 \right|
$$

$$
= \left| (2x + 5)^2 \right|
$$

and then determine the values of $x$ for which $L < 1$.

It remains to solve the quadratic inequality

$$
(2x + 5)^2 < 1,
$$

or equivalently $(2x + 5)^2 - 1 < 0$.

Taking a graphical approach, let $f(x) = (2x + 5)^2 - 1$ and find the values of $x$ for which $f(x) < 0$. Note that $f(x) = 0$ when $x = -3, -2$ and its graph is an “open-upward” parabola, thus $f(x) < 0$ when $-3 < x < -2$.

Thus the radius of convergence is $R = \frac{1}{2}$ and the series converges on the interval $(-3, -2)$.