Example

Find the 3rd degree Maclaurin polynomial (the Taylor polynomial of degree 3 at point $x = 0$) generated by the function $f(x) = e^{5x}$. Also find the general nth term of Maclaurin series (excluding zero terms and assuming the Maclaurin series begins at $n = 0$).

The Taylor polynomial $T_3$ of degree 3 of a function $f(x)$ about the point $x = 0$ is given by

$$T_3(x) = f(0) + f'(0)(x - 0) + f''(0)(x - 0)^2 + \frac{f'''(0)(x - 0)^3}{3!}.$$

First compute the appropriate derivatives. If $f(x) = e^{5x}$, then

$$f'(x) = 5e^{5x}$$
$$f''(x) = 25e^{5x}$$
$$f'''(x) = 125e^{5x}$$

using chain rule. Evaluating at $x = 0$ for use in the formula above,

$$f(0) = 1 \quad f'(0) = 5 \quad f''(0) = 25 \quad f'''(0) = 125.$$

Thus,

$$T_3(x) = 1 + 5x + \frac{25x^2}{2!} + \frac{125x^3}{3!}.$$

Notice that $f^{(n)}(0) = 5^n$, for $n = 0, 1, 2, \ldots$

It follows that the (non-zero) nth term of the Maclaurin series $\sum_{n=0}^{\infty} f^{(n)}(0) \cdot \frac{x^n}{n!}$ is

$$\frac{5^n x^n}{n!}.$$