Example

Find the first 3 nonzero terms of the Taylor series at \( x = 0 \) of

\[
f(x) = \frac{3}{1 - (x/2)}, \quad |x| < 2.
\]

First notice that \( \frac{1}{1 - (x/2)} \) is the sum of a geometric series \( \sum_{n=0}^{\infty} r^n \) where \( r = \frac{x}{2} \).

(Note that the series converges when \( |r| < 1 \), i.e. when \( |x| < 2 \).)

Thus for \( |x| < 2 \),

\[
\frac{1}{1 - (x/2)} = \sum_{n=0}^{\infty} \left( \frac{x}{2} \right)^n.
\]

Multiplying through by 3,

\[
\frac{3}{1 - (x/2)} = 3 \sum_{n=0}^{\infty} \left( \frac{x}{2} \right)^n
\]

\[
= 3 \left[ 1 + \left( \frac{x}{2} \right)^1 + \left( \frac{x}{2} \right)^2 + \left( \frac{x}{2} \right)^3 + \cdots \right]
\]

\[
= 3 + 3 \left( \frac{x}{2} \right)^1 + 3 \left( \frac{x}{2} \right)^2 + 3 \left( \frac{x}{2} \right)^3 + \cdots,
\]

thus

1st non-zero term \( = 3 \)

2nd non-zero term \( = 3 \left( \frac{x}{2} \right)^1 \)

3rd non-zero term \( = 3 \left( \frac{x}{2} \right)^2 \).