Example

Find the first 4 nonzero terms of the Taylor series for the function

\[ f(x) = \frac{7}{(1 - x)^2}, \quad |x| < 1. \]

First notice that \( \frac{1}{1-x} \) is the sum of the geometric series \( \sum_{k=0}^{\infty} x^k \). (Recall that the series converges when \( |x| < 1 \).)

So for \( |x| < 1 \),

\[ \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \]

and differentiating term by term,

\[
\frac{d}{dx} \left( \frac{1}{1-x} \right) = \frac{d}{dx} \left( \sum_{k=0}^{\infty} x^k \right)
\]

\[
\frac{1}{(1-x)^2} = \frac{d}{dx} \left( 1 + x + x^2 + x^3 + \cdots \right)
\]

\[ = 0 + 1 + 2x + 3x^2 + 4x^3 + \cdots. \]

Multiplying through by 7,

\[ \frac{7}{(1-x)^2} = 7 + 14x + 21x^2 + 28x^3 + \cdots, \]

so we have

1st non-zero term \( = 7 \)

2nd non-zero term \( = 14x \)

3rd non-zero term \( = 21x^2. \)

4th non-zero term \( = 28x^3. \)