Example

Use substitution to find the first 4 nonzero terms of the Taylor series at $x = 0$ of $f(x) = \sin \sqrt{x + 1}$.

Note that the Taylor series of $f(x) = \sin x$ at $x = 0$ is given by

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots.$$ 

Substituting $\sqrt{x + 1}$ for $x$,

$$\sin \sqrt{x + 1} = \sum_{n=0}^{\infty} \frac{(-1)^n (\sqrt{x + 1})^{2n+1}}{(2n+1)!}$$

$$= \sqrt{x + 1} - \frac{(\sqrt{x + 1})^3}{3!} + \frac{(\sqrt{x + 1})^5}{5!} - \frac{(\sqrt{x + 1})^7}{7!} + \cdots,$$

thus

1st non-zero term $= \sqrt{x + 1}$

2nd non-zero term $= -\frac{(\sqrt{x + 1})^3}{3!}$

3rd non-zero term $= \frac{(\sqrt{x + 1})^5}{5!}$

4th non-zero term $= -\frac{(\sqrt{x + 1})^7}{7!}$