Example

Consider the region of the $x$-$y$ plane bounded by the parabola $y = 2x^2$ and the lines $y = 0$, and $x = \sqrt{2}$. What is the volume of the solid obtained by rotating this region about the $x$-axis? See figure below (not drawn to scale!).

(a) **Find the area of a slice.**
Since the region borders the axis of rotation, each slice will be a disk with vertical radius (perpendicular to the axis of rotation $x$-axis.) For fixed $x$, the radius will be just the function value, or $r(x) = 2x^2$. The area of a slice is given by
\[ A(x) = \pi r^2(x) = \pi (2x^2)^2 = 4\pi x^4. \]

(b) **Find the limits of integration.**
The disk runs from $x = 0$ to $x = \sqrt{2}$ as can be seen from the above graph.

(c) **Find the volume of the region.**
The volume $V$ is given by
\[ V = \int_0^{\sqrt{2}} 4\pi x^4 \, dx = 4\pi \left( \frac{x^5}{5} \right) \bigg|_0^{\sqrt{2}} = \frac{16\sqrt{2}\pi}{5}. \]