Example

Find $\frac{dy}{dx}$ if $\sin(y) = e^y \ln(x)$.

Since we cannot solve explicitly for $y$ in terms of $x$ (note that $y = y(x)$), we use implicit differentiation. First differentiate both sides of the equation with respect to $x$:

$$\frac{d}{dx}(\sin(y)) = \frac{d}{dx}(e^y \ln(x)),$$

$$\cos(y) \cdot \frac{dy}{dx} = \frac{d}{dx}(e^y \cdot \ln(x)) + e^y \cdot \frac{d}{dx}(\ln(x))$$

$$\cos(y) \cdot \frac{dy}{dx} = e^y \cdot \frac{dy}{dx} \cdot \ln(x) + e^y \cdot \frac{1}{x}.$$

Then solve for $\frac{dy}{dx}$:

$$\frac{d}{dx}(\cos(y) - e^y \ln(x)) = \frac{e^y}{x}$$

$$\frac{dy}{dx} = \frac{e^y}{x(\cos(y) - e^y \ln(x))}.$$