Example

A homicide victim was found in a room kept at constant at 70°F. Measurements of the victim’s temperature were made once the police arrived and one hour later. The results were 82°F and 76°F respectively. Assuming Newton’s Law of Cooling and that the victim’s temperature was 99°F just before death, what was the time of death relative to the arrival of the police?

We start with Newton’s Law of Cooling

\[ \frac{dT}{dt} = k(T - T_S), \]

where \( T_S \) is the surrounding temperature. If we let \( y = T - T_S \), then \( y(0) = T(0) - T_S \), so \( y \) satisfies

\[ \frac{dy}{dt} = ky, \quad y(0) = T(0) - T_S, \]

and using a theorem from this section of the textbook we have

\[ y(t) = y(0)e^{kt} = (T(0) - T_S)e^{kt}. \]

Substituting back, we obtain the model

\[ T(t) - T_S = (T(0) - T_S)e^{kt} \]

where \( T \) is the victim’s temperature in °F as a function of time \( t \) in hours since the police arrived, surrounding temperature \( T_S = 70 \), initial temperature \( T(0) = 82 \), and constant \( k \). Substituting we have

\[ T(t) = 12e^{kt} + 70. \]

1. **Find the constant** \( k \). Using \( T(1) = 76 \)

\[
\begin{align*}
76 &= 12e^k + 70, \\
\ln\left(\frac{1}{2}\right) &= k,
\end{align*}
\]

and so

\[ k = -\ln 2. \]

2. **Use the model to determine the answer.** Using the exact value of \( k \) throughout the remainder of the problem, the model becomes

\[ T(t) = 12e^{-(\ln 2)t} + 70. \]
We use the model and estimate the time of death:

\[ \frac{99}{29} = 12e^{-(\ln 2)t} + 70 \]
\[ \frac{29}{12} = e^{-(\ln 2)t} \]

and so

\[ t = \frac{-\ln \frac{29}{12}}{\ln 2} \]

(Note that \( t \approx -1.27302 \) which implies that the time of death was about 1.27 hours prior to the arrival of the police).