Example

Compute

\[ \lim_{x \to \infty} \frac{2 \cos x + x}{e^x}. \]

Which function grows faster, \(2 \cos x + x\) or \(e^x\)?

1. **Compute the limit.**
   
   Taking the limit as \(x\) approached infinity, we arrive at the indeterminate form \(\frac{\infty}{\infty}\).
   
   Applying l’Hopital’s rule, we have
   
   \[ \lim_{x \to \infty} \frac{2 \cos x + x}{e^x} = \lim_{x \to \infty} \frac{-2 \sin x + 1}{e^x}. \]
   
   Recall now that \(\sin x\) has range \([-1, 1]\), so the numerator is always bounded above by 3 and below by \(-1\), thus
   
   \[ \frac{-1}{e^x} \leq \frac{-2 \sin x + 1}{e^x} \leq \frac{3}{e^x}. \]
   
   Since
   
   \[ \lim_{x \to \infty} \frac{-1}{e^x} = 0 = \lim_{x \to \infty} \frac{3}{e^x}, \]
   
   we can apply Sandwich theorem to conclude that
   
   \[ \lim_{x \to \infty} \frac{2 \cos x + x}{e^x} = 0. \]

2. **Determine which function grows faster.** We have

   \[ \lim_{x \to \infty} \frac{f(x)}{g(x)} = 0 \]

   where \(f(x) = \cos x + x\) and \(g(x) = e^x\), so \(f\) grows slower than \(g\) (i.e. \(g\) grows faster than \(f\)).