Example

Evaluate
\[ \int \frac{x + 1}{x - 1} \, dx. \]

There are two ways to approach this problem.

1. **Use long division.**
   The integrand is an improper fraction, so using long division
   \[ \int \frac{x + 1}{x - 1} \, dx = \int 1 + \frac{2}{x - 1} \, dx. \]
   Make the substitution \( u = x - 1 \) \((du = dx)\) into the second term to obtain
   \[ \int \frac{x + 1}{x - 1} \, dx = \int dx + 2 \int \frac{du}{u} = x + 2 \ln |x - 1| + C. \]

2. **Add and subtract one in the numerator, then split the integrand.**
   Note that
   \[ \frac{x + 1}{x - 1} = \frac{x + (-1 + 1) + 1}{x - 1} = \frac{(x - 1) + 2}{x - 1} = \frac{x - 1}{x - 1} + \frac{2}{x - 1} = 1 + \frac{2}{x - 1}. \]
   The rest of the problem is carried out the same as above.