1. (to accompany problem #1) Suppose a solid lies between the planes perpendicular to the \( x \)-axis at \( x = -2 \) and \( x = 3 \). The cross sections perpendicular to the \( x \)-axis and these planes run from \( y = -\sqrt{x + 2} \) to \( y = \sqrt{x + 2} \). Find the volume of the solid if the cross sections are squares with diagonals in the \( xy \)-plane.

(a) Find the area of a cross section.
Since the cross sections are squares whose diagonal \( d \) runs from \( y = -\sqrt{x + 2} \) to \( y = \sqrt{x + 2} \), it follows that

\[
    d = \sqrt{x + 2} - (-\sqrt{x + 2}) = 2\sqrt{x + 2}.
\]

Recalling the relationship between the side \( s \) of a square and its diagonal \( d \), we can use either pythagorean theorem \( s^2 + s^2 = d^2 \), so that \( 2s^2 = d^2 \), or one can use right triangle trigonometry to see that \( \cos(45^\circ) = \frac{s}{d} \), i.e. \( \frac{\sqrt{2}}{2} d = s \). Thus the area of a cross section is given by

\[
    A(x) = s^2 = (\sqrt{2}\sqrt{x + 2})^2 = 2(x + 2)
\]

(b) Find the volume of the region.
Since the cross sections are stacked along the \( x \)-axis from \( x = -2 \) to \( x = 3 \), the volume \( V \) is just the integral of the area \( A(x) \) from \( x = -2 \) to \( x = 3 \), i.e.

\[
    V = \int_{-2}^{3} 2(x + 2)\,dx = (x^2 + 4x)\Big|_{-2}^{3} = 21 - (-4) = 25.
\]
2. (to accompany problems #4-8) Find the volume of the solid generated by rotating the region bounded by $y = x^2$, $y = 1$, and $x = 4$ about the line $y = 1$. See figure below (not drawn to scale!)

(a) Find the area of a slice.
   Since the region borders the axis of rotation, each slice will be a disk with vertical radius (perpendicular to the axis of rotation $y = 1$.) For fixed $x$, the radius will be the top curve minus the bottom, or $r(x) = x^2 - 1$. The area of a slice is given by
   
   $$A(x) = \pi r(x)^2 = \pi (x^2 - 1)^2.$$ 

(b) Find the limits of integration.
   The disks run from the $x$-coordinate of the intersection of $y = x^2$ with $y = 1$ in the first quadrant, so from $x = 1$, to $x = 4$.

(c) Find the volume of the region.
   The volume $V$ is given by
   
   $$V = \int_1^4 \pi (x^2 - 1)^2 \, dx = \pi \left[ \frac{x^5}{5} - \frac{2x^3}{3} + x \right]^4_1 = \frac{828\pi}{5}$$