Example

Evaluate the following limit:

$$\lim_{x \to 0^+} x^2 \ln x.$$

Solution: As $x \to 0^+$, $x^2 \to 0$ and $\ln x \to -\infty$. Thus we are dealing with an indeterminate
which (up to sign) is of the form $0 \cdot \infty$.

Writing $x^2 \ln x$ as

$$\frac{\ln x}{1/x^2},$$

we can apply L’Hopital’s rule $\infty/\infty$:

$$\lim_{x \to 0^+} x^2 \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x^2} = \lim_{x \to 0^+} \frac{1/x}{-2/x^3} = - \lim_{x \to 0^+} \frac{x^2}{2} = 0.$$

Therefore

$$\lim_{x \to 0^+} x^2 \ln x = 0.$$