Example

Sketch the graph of the curve

\[ r^2 = 2 \cos \theta. \]

Note that if the point \([r, \theta]\) in polar coordinates lies on the graph of \(r^2 = 2 \cos \theta\), then

- \([r, -\theta]\) lies on the graph since \(2 \cos(-\theta) = 2 \cos \theta = r^2\), so the curve is symmetric about the x-axis.
- \([-r, \theta]\) lies on the graph since \((-r)^2 = r^2\), so the curve is symmetric about the origin.
- from the two previous symmetries it follows that \([-r, -\theta]\) lies on the graph since \(2 \cos(-\theta) = 2 \cos \theta = r^2 = (-r)^2\), so the curve is symmetric about the y-axis.

We plot the curve by hand by constructing a table of values \([r, \theta]\) evaluating the polar curve at increasing values of \(\theta\) starting at \(\theta = 0\). The approximate values of \(r\) in the table are obtained using the (approximate) values of the cosine function found in the handout *Sine and Cosine for Standard Angles* (linked to some of the problems in this WeBWorK section).

Since \(r^2 = 2 \cos \theta\) implies \(\cos \theta\) must be positive, the entire graph can be traced out for \(\theta \in [0, \pi/2] \cup [3\pi/2, 2\pi]\). Due to symmetry, we first consider \(\theta \in [0, \pi/2]\).

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>0</th>
<th>(\pi/6)</th>
<th>(\pi/4)</th>
<th>(\pi/3)</th>
<th>(\pi/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = \pm \sqrt{2 \cos \theta})</td>
<td>(-\sqrt{2}, \sqrt{2})</td>
<td>(-\sqrt{3}, \sqrt{3})</td>
<td>(-\sqrt{2}, \sqrt{2})</td>
<td>(-1, 1)</td>
<td>0</td>
</tr>
<tr>
<td>approx. (r)</td>
<td>(-1.4, 1.4)</td>
<td>(-1.3, 1.3)</td>
<td>(-1.2, 1.2)</td>
<td>(-1, 1)</td>
<td>0</td>
</tr>
</tbody>
</table>

Using symmetry, we obtain the complete graph for \(\theta \in [0, \pi/2] \cup [3\pi/2, 2\pi]\). (See next page.)
Using the page of graphs, we label this curve as 2H (lemniscate horizontal). Although this information is not needed for every problem, we note that the point $P$ corresponding to the angle $\theta = \frac{\pi}{2}$ has polar coordinates $[r, \theta] = [0, \pi/2]$, and the point $Q$ corresponding to the angle $\theta = 0$ has polar coordinates $[r, \theta] = [\sqrt{2}, 0]$. 