Example

Let \( a_k = \frac{5}{12^k} \).
Is the sequence \( \{a_k\} \) convergent or divergent? If it is convergent, find its limit.

Is the series \( \sum_{k=1}^{\infty} a_k \) convergent or divergent? If it is convergent, find its sum.

1. Determine the limit of the sequence.

\[
\lim_{n \to \infty} \frac{5}{12^k} = 5 \lim_{k \to \infty} \left( \frac{1}{12} \right)^k = 5 \cdot 0 = 0.
\]

The sequence converges and its limit is 0.

2. Determine convergence of the series.

By part a), the \( k \)th term test (for divergence) is inconclusive. However, the series

\[
\sum_{k=1}^{\infty} \frac{5}{12^k} = 5 \sum_{k=0}^{\infty} \left( \frac{1}{12} \right)^{k+1} = \frac{5}{12} \sum_{k=0}^{\infty} \left( \frac{1}{12} \right)^k
\]

is a geometric series with \( x = \frac{1}{12} \) which is convergent and its sum is given by

\[
\sum_{k=1}^{\infty} \frac{5}{12^k} = \frac{5}{12} \cdot \frac{1}{1 - (1/12)} = \frac{5}{11}.
\]