Example

Consider the series
\[ \sum_{k=1}^{\infty} (-1)^k \frac{(k + 1)!^3}{e^k 8k}. \]

First use the ratio test to determine whether or not the series converges absolutely. If the ratio test is inconclusive, can the answer be found using another test?

Solution: Computing the limit of the ratio,

\[
\bar{\lambda} = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \to \infty} \left| \frac{(-1)^{k+1}((k + 2)!)^3}{e^{k+1} 8(k + 1)} \cdot \frac{e^k 8k}{(-1)^k((k + 1)!)^3} \right|
\]

\[= \lim_{k \to \infty} \left| \frac{(-1)((k + 2)!)^3}{e(k + 1)} \cdot \frac{k}{((k + 1)!)^3} \right|
\]

\[= \lim_{k \to \infty} \left| \frac{(k + 2)!^3}{(k + 1)!^3} \cdot \frac{k}{e(k + 1)} \right|
\]

\[= \lim_{k \to \infty} \left| \frac{(k + 2)(k + 1)!}{(k + 3)!} \cdot \frac{k}{e(k + 1)} \right|
\]

\[= \lim_{k \to \infty} \left| \frac{(k + 2)^3 k}{e(k + 1)} \right| = \infty.
\]

We conclude by the ratio test that the series does NOT converge absolutely.