Example

Find the interval of convergence for the series

\[
\sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k + 2}.
\]

Do not check convergence at the endpoints of intervals.

We first compute

\[
\bar{\lambda} = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right|
\]

\[
= \lim_{k \to \infty} \left| \frac{(-1)^{k+1} x^{k+1} / (k + 3)}{(-1)^k x^k / (k + 2)} \right|
\]

\[
= \lim_{k \to \infty} \left| (-1) \cdot \frac{k + 2}{k + 3} \cdot x \right|
\]

\[
= |x|,
\]

and then determine the values of \( x \) for which \( \bar{\lambda} < 1 \). It follows that

\[
|x| < 1 \iff -1 < x < 1,
\]

thus the series converges on the interval \((-1, 1)\).