Example

Use substitution to find the first 4 nonzero terms of the Taylor series of

\[ f(x) = \sin \sqrt{x}. \]

Note that the Taylor series of \( f(x) = \sin x \) is given by

\[
\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots.
\]

Substituting \( \sqrt{x} \) for \( x \),

\[
\sin \sqrt{x} = \sum_{k=0}^{\infty} \frac{(-1)^n (\sqrt{x})^{2k+1}}{(2k+1)!}
= \sqrt{x} - \frac{(\sqrt{x})^3}{3!} + \frac{(\sqrt{x})^5}{5!} - \frac{(\sqrt{x})^7}{7!} + \cdots,
\]

thus

1st non-zero term \( = \sqrt{x} \)

2nd non-zero term \( = -\frac{(\sqrt{x})^3}{3!} \)

3rd non-zero term \( = \frac{(\sqrt{x})^5}{5!} \)

4th non-zero term \( = -\frac{(\sqrt{x})^7}{7!} \)