Example

It took 45 J of work to stretch a spring from its natural length of 20 cm to a length of 1 m. Find the spring’s force constant.

(a) **Convert length measurements to meters.**
We will measure work in Joules (J), force in Newtons (N), and displacement in meters (m) in order to employ the relation $J = N \cdot m$. Since 1 m = 100 cm, it follows that

$$20 \text{ cm} = 20 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = \frac{1}{5} \text{ m}.$$

(b) **Use the formula for work and Hooke’s Law to find $k$.**
According to Hooke’s Law, the force $F$ is modeled using $F(x) = kx$. Since we wish to stretch the spring $z = 1 - \frac{1}{5} = \frac{4}{5}$ m beyond its natural length, we have that the work $W$ satisfies

$$W = \int_{0}^{z} F(x) \, dx = \int_{0}^{4/5} kx \, dx = 45.$$

Solving for $k$ in

$$45 = \frac{kx^2}{2} \bigg|_{0}^{4/5} = \frac{k(4/5)^2}{2} = \frac{8}{25} k,$$

it follows that

$$k = \frac{45}{8/25} = \frac{1125}{8}.$$