Example

Let \( f(x) = (x - 3)^2 - 6, x \geq 3 \). Find the value of \( \frac{df^{-1}}{dx} \) at the point \( x = -2 = f(5) \).

By the Inverse Function Theorem,

\[
\left. \frac{df^{-1}}{dx} \right|_{x=-2} = (f^{-1})'(-2) = \frac{1}{f'(f^{-1}(-2))},
\]

(provided \( f'(f^{-1}(-2)) \neq 0 \)).

From \( f(5) = -2 \) it follows that \( f^{-1}(-2) = 5 \), and with \( f'(x) = 2x - 6 \),

\[
(f^{-1})'(-2) = \frac{1}{f'(5)} = \frac{1}{4}.
\]