Example

Evaluate the following:

(i) \( \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \)

(ii) \( \tan^{-1}(\sqrt{3}) \)

(iii) \( \sec^{-1}(1) \)

Solution:

(i) The quantity \( \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \), equivalently, \( \arcsin\left(-\frac{\sqrt{3}}{2}\right) \), is that angle \( \theta \in [-\pi/2, \pi/2] \) for which

\[
\sin \theta = -\frac{\sqrt{3}}{2}.
\]

But for \( \theta = -\pi/3 \) it follows that \( \sin \theta = -\sqrt{3}/2 \), so

\[
\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}.
\]

(ii) \( \tan^{-1}(\sqrt{3}) \) is that angle \( \theta \in (-\pi/2, \pi/2) \) for which

\[
\tan \theta = \sqrt{3}.
\]

But \( \tan(\pi/3) = \sqrt{3} \) so it follows that

\[
\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}.
\]

(iii) To evaluate \( \sec^{-1}(1) = \arccos^{-1}(1) \), one must determine the angle \( \theta \in [0, \pi/2) \cup (\pi/2, \pi] \) which uniquely satisfies

\[
\sec(\theta) = 1, \quad \text{or} \quad \cos(\theta) = \frac{1}{1} = 1.
\]

Since \( \cos(0) = 1 \), it follows that

\[
\sec^{-1}(1) = 0.
\]

Note: For an easy way to remember the trigonometric functions at standard angles, see the document *Notes on sine and cosine functions* which is linked to the statement of the WeBWorK problem.