Example

Find the interval of convergence for the following series

\[-2 \ln (2)x + 4 \ln (3)x^2 - 8 \ln (4)x^3 + 16 \ln (5)x^4 - 32 \ln (6)x^5 + \cdots.\]

Do not check convergence at the endpoints of intervals.

Solution: From the series, we can get the general form of \( a_n = (-2)^n \ln (n + 1)x^n \), for \( n \geq 1 \), and then apply the ratio test, we first compute

\[
\rho = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
\]

\[
= \lim_{n \to \infty} \left| \frac{(-2)^{n+1} \ln (n + 2)x^{n+1}}{(-2)^n \ln (n + 1)x^n} \right|
\]

\[
= \lim_{n \to \infty} \left| (-2) \cdot \frac{\ln (n + 2)}{\ln (n + 1)} \cdot x \right|
\]

\[
= |2x|,
\]

and then determine the values of \( x \) for which \( \rho < 1 \). It follows that

\[
|2x| < 1 \iff -\frac{1}{2} < x < \frac{1}{2},
\]

thus the series converges on the interval \((-\frac{1}{2}, \frac{1}{2})\).