1. Find the vector equation of the line in which the 2 planes $2x - 5y + 3z = 12$ and $3x + 4y - 3z = 6$ meet.

**Solution:** The vector $\langle 2, -5, 3 \rangle$ is normal (i.e. perpendicular) to the plane $2x - 5y + 3z = 12$. The vector $\langle 3, 4, -3 \rangle$ is normal to the plane $3x + 4y - 3z = 6$. These vectors aren’t parallel so the planes do meet!

Now, the vector or cross product of these two normal vectors gives a vector which is perpendicular to both of them and which is therefore parallel to the line of intersection of the two planes. So this cross product will give a direction vector for the line of intersection.

The cross product of $\langle 2, -5, 3 \rangle$ and $\langle 3, 4, -3 \rangle$ is $\langle 3, 15, 23 \rangle$.

In order to find the vector equation of the line of intersection, we also need to find the position vector from the origin of some point which lies on it. So we need to find some point which lies on both the planes because then it must lie on their line of intersection. Any point which lies on both planes will do.

I can see that both planes will have points for which $x = 0$.

These points in $2x - 5y + 3z = 12$ will have $-5y + 3z = 12$.

These points in $3x + 4y - 3z = 6$ will have $4y - 3z = 6$.

Solving these two equations simultaneously gives $y = -18$ and $z = -26$ so the point with position vector $(0, -18, -26)$ lies on the line of intersection.

Therefore the equation of the line of intersection is

$$r = (0, -18, -26) + t\langle 3, 15, 23 \rangle$$

Check for yourself that if you choose some value for $t$ (say $t = 2$) that the point that you get does really lie on both planes and so on their line of intersection.

Let $\langle 2, -5, 3 \rangle$ be vector $A$, $\langle 3, 4, -3 \rangle$ vector $B$.

By using dot product we have

$$\cos \theta = \frac{A \cdot B}{|A| \cdot |B|} = \frac{-23}{\sqrt{38} \cdot \sqrt{34}} = \frac{-23}{\sqrt{1292}}$$

So the angle is:

$$\theta = \cos^{-1} \left( \frac{-23}{\sqrt{1292}} \right)$$