1. Solve the initial value problem: \( \mathbf{r}'(t) = (2t + 6t^2, e^t, 1 + \sin(t)) \) and \( \mathbf{r}(0) = (5, 1, 5) \)

**Solution:**

First each component of the vector function \( \mathbf{r}'(t) \) is integrated with respect to \( t \), obtaining:

\[
\mathbf{r}(t) = (t^2 + 2t^3 + a, e^t + b, t + \cos(t) + c)
\]

The initial condition is then used to solve for each of the constants \( a, b, c \) by evaluating the recently found \( \mathbf{r}(t) \) at the time of the initial condition (in this case \( t = 0 \)).

\[
\mathbf{r}(0) = (0^2 + 2(0)^3 + a, e^0 + b, (0) + \cos(0) + c) = (5, 1, 5)
\]

So:

\[
\begin{align*}
0 + 0 + a &= 5 \implies a = 5, \\
1 + b &= 1 \implies b = 0, \\
0 + 1 + c &= 5 \implies c = 4
\end{align*}
\]

Therefore \( \mathbf{r}(t) = (t^2 + 2t^3 + 5, e^t, t + \cos(t) + 4) \)