1. A projectile is fired with muzzle speed 150 m/s and angle of elevation 45° from a position 10 m above ground level. Where does the projectile hit the ground and with what speed?

**Solution:** If we place the origin at ground level, then the initial position of the projectile is (0, 10) and so the parametric equation of the trajectory are therefore

\[ x = (v_0 \cos \alpha) t \]
\[ y = 10 + (v_0 \sin \alpha) t - \frac{1}{2} gt^2 \]

With \( v_0 = 150 \text{ m/s}, \ \alpha = 45°, \ \text{and } g = 9.8 \text{ m/s}^2 \), we have

\[ x = 150 \cos \left( \frac{\pi}{4} \right) t = 75\sqrt{2} t \]
\[ y = 10 + 150 \sin \left( \frac{\pi}{4} \right) t - \frac{1}{2} (9.8) t^2 = 10 + 75\sqrt{2} t - 4.9t^2 \]

Impact occurs when \( y = 0 \), that is, \( 4.9t^2 - 75\sqrt{2}t - 10 = 0 \). Solving this quadratic equation (and using only the positive value of t), we get

\[ t = \frac{-75\sqrt{2} + \sqrt{11,250 + 196}}{9.8} \approx 21.74 \]

Then \( x \approx 75\sqrt{2}(21.74) \approx 2306 \), so the projectile hits the ground about 2306 m away. The velocity of the projectile is

\[ \mathbf{v}(t) = \mathbf{r}'(t) = 75\sqrt{2}\mathbf{i} + (75\sqrt{2} - 9.8t)\mathbf{j} \]

So its speed at impact is

\[ | \mathbf{v}(21.74) | = \sqrt{(75\sqrt{2})^2 + (75\sqrt{2} - 9.8 \cdot 21.74)^2} \approx 151 \text{ m/s} \]