1. Show that the function:

\[ g(x, y, z) = \begin{cases} 
\frac{xy + yz}{x^2 + y^2 + z^2} & \text{if } (x, y, z) \neq (0, 0, 0) \\
0 & \text{if } (x, y, z) = (0, 0, 0) 
\end{cases} \]

is not continuous at \((0, 0, 0)\).

**Solution:** To do this we will show that:

\[ \lim_{(x,y,z)\to(0,0,0)} g(x, y, z) \neq g(0, 0, 0) \]

We select the path:

In the plane \(z = 0\) along the line \(y = x\).

which shows:

\[ \lim_{(x,x,0)\to(0,0,0)} g(x, y, z) = \lim_{(x,x,0)\to(0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2} = \lim_{(x,x,0)\to(0,0,0)} \left[ \frac{1}{2} \right] \]

Evaluate the limit we get:

\[ \lim_{(x,x,0)\to(0,0,0)} \frac{xy + yz}{x^2 + y^2 + z^2} = \frac{1}{2} \]

Since \(\frac{1}{2} \neq 0\), we conclude that \(g(x, y, z)\) is not continuous at \((0, 0, 0)\).