1. Find the limit

\[ \lim_{(x,y) \to (0,0)} (x^2 + y^2) \cos(x^2 + y^2) \]

(a) 0

Solution: Since

\[-1 \leq \cos(x^2 + y^2) \leq 1\]

we have

\[-(x^2 + y^2) \leq (x^2 + y^2) \cos(x^2 + y^2) \leq x^2 + y^2\]

Now since both

\[ \lim_{(x,y) \to (0,0)} -(x^2 + y^2) = 0 \]

and

\[ \lim_{(x,y) \to (0,0)} (x^2 + y^2) = 0 \]

We must have

\[ \lim_{(x,y) \to (0,0)} (x^2 + y^2) \cos(x^2 + y^2) = 0 \]