1. Find the directional derivative of the function $f(x, y) = x^2y^3 - 4y$ at the point $(2, -1)$ in the direction of the vector $v = (2, 5)$.

**Solution:** We first compute the gradient vector at $(2, -1)$:

$$
\nabla f(x, y) = 2xy^3i + (3x^2y^2 - 4)j
$$

$$
\nabla f(2, -1) = -4i + 8j
$$

Note that $v$ is not a unit vector, but since $|v| = \sqrt{29}$, the unit vector in the direction of $v$ is

$$
u = \frac{v}{|v|} = \frac{2}{\sqrt{29}}i + \frac{5}{\sqrt{29}}j
$$

Therefore,

$$
Du f(2, -1) = \nabla f(2, -1) \cdot u = (-4i + 8j) \cdot \left( \frac{2}{\sqrt{29}}i + \frac{5}{\sqrt{29}}j \right)
$$

$$
= \frac{-4 \cdot 2 + 8 \cdot 5}{\sqrt{29}} = \frac{32}{\sqrt{29}}
$$