1. Evaluate \( \int\int_{R}(3x^2 + 4y^2)dA \), where \( R \) is the region in the upper half-plane bounded by the circles \( x^2 = y^2 = 1 \) and \( x^2 + y^2 = 4 \).

**Solution:** The region can be described as:

\[
R = \{(x, y) | y \geq 0, 1 \leq x^2 + y^2 \leq 4\}
\]  

(1)

It is the half ring given in polar coordinates by \( 1 \leq r \leq 2, 0 \leq \theta \leq \pi \). Therefore we have:

\[
\int\int_{R}(3x^2 + 4y^2)dA = \int\int_{R}(3r^2 + 3y^2) + y^2 dA
\]

\[
= \int_{0}^{\pi} \int_{1}^{2} (3r^2 + (r \sin \theta)^2) r dr d\theta
\]

\[
= \int_{0}^{\pi} \int_{1}^{2} 3r^3 + r^3 \sin^2 \theta dr d\theta
\]

\[
= \int_{0}^{\pi} \left[ \frac{3}{4}r^4 + \frac{1}{4}r^4 \sin^2 \theta \right]_{r=1}^{r=2} d\theta
\]

\[
= \int_{0}^{\pi} (12 + 4 \sin^2 \theta) - \left( \frac{3}{4} + \frac{1}{4} \sin^2 \theta \right) d\theta
\]

\[
= \int_{0}^{\pi} \frac{45}{4} \sin^2 \theta d\theta + \int_{0}^{\pi} \frac{15}{4} (1 - \cos 2\theta) d\theta
\]

\[
= \left[ \frac{45}{4} \theta + \frac{15}{8} \sin 2\theta - \frac{15}{16} \sin 2\theta \right]_{0}^{\pi}
\]

\[
= \frac{105}{8} \pi
\]