1. Determine the volume of the region that lies under the sphere \( x^2 + y^2 + z^2 = 9 \), above the plane \( z = 0 \) and inside the cylinder \( x^2 + y^2 = 5 \).

**Solution:** We know that the formula for finding the volume of a region is,

\[
V = \iiint_D f(x,y)\,dA
\]

In order to make use of this formula we’re going to need to determine the function that we should be integrating and the region \( D \) that we’re going to be integrating over.

The function isn’t too bad. It’s just the sphere, however, we do need it to be in the form \( z = f(x,y) \). We are looking at the region that lies under the sphere and above the plane \( z = 0 \) (just the \( xy \)-plane right?) and so all we need to do is solve the equation for \( z \) and when taking the square root we’ll take the positive one since we are wanting the region above the \( xy \)-plane. Here is the function.

\[
z = \sqrt{9 - x^2 - y^2}
\]

The region \( D \) isn’t too bad in this case either. As we take points \( (x,y) \), from the region we need to completely graph the portion of the sphere that we are working with. Since we only want the portion of the sphere that actually lies inside the cylinder given by \( x^2 + y^2 = 5 \) this is also the region \( D \). The region \( D \) is the disk \( x^2 + y^2 \leq 5 \) in the \( xy \)-plane.

So, the region that we want the volume for is really a cylinder with a cap that comes from the sphere.

We are definitely going to want to do this integral in terms of polar coordinates so here are the limits (in polar coordinates) for the region,

\[
0 \leq \theta \leq 2\pi \\
0 \leq r \leq \sqrt{5}
\]

and we’ll need to convert the function to polar coordinates as well.

\[
z = \sqrt{9 - x^2 - y^2} = \sqrt{9 - r^2}
\]

The volume is then,

\[
V = \iint_D \sqrt{9 - x^2 - y^2}\,dA \\
= \int_0^{2\pi} \int_0^{\sqrt{5}} r\sqrt{9 - r^2}\,dr\,d\theta \\
= \int_0^{2\pi} \left[ \frac{1}{3} \right]_{\sqrt{5}}^{\sqrt{9}} (9 - r^2)^{\frac{3}{2}} \,d\theta \\
= \int_0^{2\pi} \frac{19}{3} \,d\theta \\
= \frac{38\pi}{3}
\]