1. Convert the point \((-1, 1, -\sqrt{2})\) from Cartesian to spherical coordinates.

**Solution:** The first thing that we do here is find \(\rho\).

\[
\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{1 + 1 + 2} = 2
\]

Now we need to find \(\varphi\). We can do this using the conversion for \(z\).

\[
z = \rho \cos \varphi \Rightarrow \cos \varphi = \frac{z}{\rho} = \frac{-\sqrt{2}}{2} \Rightarrow \varphi = \cos^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{3\pi}{4}
\]

As with the last parts this will be the only possible \(\varphi\) in the range allowed.

Finally, let’s find \(\theta\). To do this we can use the conversion for \(x\) or \(y\). We will use the conversion for \(y\) in this case.

\[
\sin \theta = \frac{y}{\rho \sin \varphi} = \frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{\pi}{4} \text{ or } \theta = \frac{3\pi}{4}
\]

Now, we actually have more possible choices for \(\theta\) but all of them will reduce down to one of the two angles above since they will just be one of these two angles with one or more complete rotations around the unit circle added on.

We will however, need to decide which one is the correct angle since only one will be. To do this let’s notice that, in two dimensions, the point with coordinates \(x = -1\) and \(y = 1\) lies in the second quadrant. This means that \(\theta\) must be an angle that will put the point into the second quadrant. Therefore, the second angle, \(\theta = \frac{3\pi}{4}\), must be the correct one.

The spherical coordinates of this point are then \((2, \frac{3\pi}{4}, \frac{3\pi}{4})\).